

*Precalculus*  
*Unit 3: Logarithmic and Inverse Functions*

*Farmington Public Schools*  
*Grade 11-12*  
*Mathematics*

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## **Unit Summary**

This unit on Logarithmic and Inverse Functions is the third unit in the College Preparatory Precalculus curriculum following the unit on Linear and Exponential Functions and preceding the unit on Polynomial and Rational Functions. This unit will introduce logarithms as a standalone function family. A connection will be made to the importance of logarithmic functions in solving exponential relations using the concept of logarithms as inverses of exponential functions. In addition, problem situations in daily class activities and on the authentic assessment will emphasize the importance of these concepts as they are needed for real-world application and modeling. The unit will last approximately 5 weeks.

## **Stage One: Standards**

### **Essential Understandings and Content Standards**

*(from: **Connecticut Mathematics Curriculum Framework – 9-12**)*

**Algebraic Reasoning: Patterns and Functions Understanding #1.1** – Students should understand and describe patterns and functional relationships. \*

#### **Content Standards:**

Students will be able to:

- **Model real-world situations and make generalizations about mathematical relationships using a variety of patterns and functions.**
  - Describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic ...
  - Analyze essential relations in a problem to determine the possible functions that could model the situation.

**Algebraic Reasoning: Patterns and Functions Understanding #1.3** – Students should use operations, properties and algebraic symbols to determine equivalence and solve problems.\*

#### **Content Standards:**

Students will be able to:

- **Use and extend algebraic concepts to include real and complex numbers, ...**
  - Combine, compose and invert functions.
  - Use logarithms ... to solve problems

**Numerical and Proportional Reasoning Understanding #2.2** – Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities. \*

#### **Content Standards:**

Students will be able to:

- **Investigate mathematical properties and operations related to objects that are not numbers.**
  - Perform operations with ... logarithms.

**Working With Data: Probability and Statistics Understanding #4.1** – Students should collect, organize and display data using appropriate statistical and graphical methods. \*

#### **Content Standards:**

Students will be able to:

- **Create the appropriate visual or graphical representation of real data.**
  - Collect real data and create meaningful graphical representations of the data.
  - Develop, use and explain applications and limitations of linear and nonlinear models and regression in a variety of contexts.

**Working With Data: Probability and Statistics Understanding #4.2** – Students should analyze data sets to form hypotheses and make predictions. \*

#### **Content Standards:**

Students will be able to:

- **Analyze real-world problems using statistical techniques.**
  - Estimate an unknown value between data points on a graph (interpolation) and make predictions by extending the graph (extrapolation).



## **Unit Outcome Statement**

As a result of this unit the students will understand the logarithmic function and its relationship as the inverse of exponential functions. In addition, students will understand inverse relationships and inverse functions as they are used to analyze, model, and solve real-world applications and problems.

Students will investigate these relationships as they relate involving measurements of real-world situations, such as earthquake intensity, audio amplification, etc.

## **Essential Questions**

*How is the concept of “undoing” essential to the study of mathematics?*

*When and why would it be beneficial to represent function families and/or data relationships by re-defining them?*

*How could the understanding of logarithmic and inverse functions be useful in modeling and analyzing real-world situations?*

## **Knowledge and Skills**

### **Knowledge**

- Properties of exponents and logarithms
- Definition of logarithm and logarithmic function
- Understand domain and range of logarithmic function
- The logarithm is the inverse of an exponential
- Understanding inverse relationships graphically and algebraically.
  - Reflection across  $y=x$
  - Composition of a function with its inverse
  - One-to-one and onto relationships

### **Skills/Processes**

- Determining a function's inverse
- Determining if an inverse is a function
- Verify inverse relationships algebraically using composition of functions
- Using logarithms as inverse of exponentials (and vice-versa) in solving problems
- Simplifying logarithmic/exponential expressions
- Using technology to obtain best-fit regression for data.
- Apply knowledge and skills to stand-alone problems and real-world situations

### **Thinking Skills**

- Using regressed model to estimate outcomes (interpolation/extrapolation)
- "Linearization" to analyze data behavior in order to better determine regression model.
- Applying process knowledge to real-world applications appropriately

## **Stage Two: Assessment Package**

### **Authentic Performance Task**

**Goal:** Your task is to convince the public health department of the state of Connecticut to actively support HIV/AIDS reduction research.

**Role:** You are a concerned high school student who is aware of the growing HIV/AIDS problem throughout the nation and the world. You decided to do something to have a positive impact in your state.

**Audience:** Information will be presented to state officials (department of health and legislative), educators, and classmates.

**Situation:** You are given data on national annual deaths from AIDS for the years 1981 through 1994 (see appendix A). Although this data is not current, you decide to use it. To present a strong argument you look at the data several ways.

- Scatter plot of annual deaths – modeling the data and determining whether or not the number of annual deaths will level out to a constant rate.
- Total number of deaths since 1981 – modeling the data and determining in what year the total number of deaths will exceed the current population of the State of Connecticut.

**Performance:** You will need to create a report of your results or be able to support its content in a presentation, if asked to do so.

- Scatter plots using supplied data
  - Annual deaths
  - Total deaths since 1981
- Analysis of data set / scatter plot determining best regression model so that predictions could be reliable
  - Annual deaths – how was regression selected and why it fits or why it does not fit using mathematical analysis.
  - Total deaths since 1981– how was regression selected and why it fits or why it does not fit
- Describe the behavior of the graph of each model
- Discuss the types of models determined and how they each relate to your goal.
- Discussion on how logarithms are used in your analysis/verification
- Discussion on how inverse relations are used in your analysis/verification

**Standards & Criteria for Success:** The standard and criteria for success will be addressed in a rubric which reflect a 4 point scale (4 = Exceeds Standard, 3 = Meets Standard, 2 = Near Standard, 1 = Below Standard)

Content standards addressed with assessment:

**Algebraic Reasoning: Patterns and Functions Understanding #1.1** – Students should understand and describe patterns and functional relationships.

**Algebraic Reasoning: Patterns and Functions Understanding #1.3** – Students should use operations, properties and algebraic symbols to determine equivalence and

solve problems.

**Numerical and Proportional Reasoning Understanding #2.2** – Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

**Working With Data: Probability and Statistics Understanding #4.1** – Students should collect, organize and display data using appropriate statistical and graphical methods.

**Working With Data: Probability and Statistics Understanding #4.2** – Students should analyze data sets to form hypotheses and make predictions.

### **Tests, Quizzes, and Other Quick and Ongoing Checks for Understanding**

Unit test (see appendix B) stresses math knowledge and skills directly with applications and modeling through a variety of multiple choice questions. This test structured in such a manner that several of the content standards and the school wide cumulative assessment are addressed.

Content standards addressed with assessment:

**Algebraic Reasoning: Patterns and Functions Understanding #1.1** – Students should understand patterns and functional relationships.

**Algebraic Reasoning: Patterns and Functions Understanding #1.3** – Students should use operations, properties and algebraic symbols to determine equivalence and solve problems.

**Numerical and Proportional Reasoning Understanding #2.2** – Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

**Working With Data: Probability and Statistics Understanding #4.2** – Students should analyze data sets to form hypotheses.

Benchmark quizzes (see appendices C and D) check for understanding of concepts, skills, and application through a variety of free response questions. The quizzes are structured in such a manner that skills needed for use with the performance task and unit test are verified.

- Quiz 1: Logarithmic Functions: Properties and Definitions  
This quiz is an assessment of the skills and properties associated with logarithms. Skill based questions will focus on being able to correctly apply the properties of exponents and logarithms to simplifying expressions. Application questions will focus on a situation that can be modeled by a logarithmic function.
- Quiz 2: Inverse Functions and Data Analysis  
This quiz is an assessment of understanding of what an inverse function is, how to find an inverse function for basic equations, and how to verify whether a function is an inverse of another function. In addition, the data analysis will be assessed through linearizing and transforming data in order to find an appropriate function model.

Progress check (Appendix E, F): These regular checks for understanding will occur throughout the unit. Some examples include:

- Animal Species Population  
Students will be given a logarithmic equation and asked to use it to make predictions on the deer population after a certain time. This particular model can be used often, just by changing what questions are asked about it.
- Inverse Functions  
Students will find the inverse of a linear function, and discuss how to verify that their function is an inverse graphically and algebraically.

### **Stage Three: Learning Experiences and Instruction**

Stage Three helps teachers plan learning experiences and instruction that align with Stage One and enables students to be successful in Stage Two.

#### **Learning Experiences and Instruction**

The learning experiences and instruction described in this section provide teachers with one option for meeting the standards listed in Stage One. Teachers are encouraged to design their own learning experiences and instruction, tailored to the needs of their particular students.

<b><i>Guiding Questions</i></b>	<b><i>Instructional Strategies</i></b>	<b><i>Checking for Understanding</i></b>
<b><i>Lesson Topic: Definition and Behavior of Logarithms</i></b>		
<p><i>How do you determine which had a bigger bang – the San Francisco earthquake or Mount Saint Helens volcano?</i></p> <p><i>How is a large number expressed as a small number?</i></p> <p><i>Why are small positive numbers expressed as negative numbers?</i></p> <p><i>Why don't negative numbers have logarithmic equivalents?</i></p> <p><b><i>How could the understanding of logarithmic and inverse functions be useful in modeling and analyzing real-world situations?</i></b></p>	<p><i>“Magnitude of Disaster”</i></p> <p><b><i>Hook:</i></b> <i>Students will be looking at real data from the Kilauea Caldera in Hawaii to begin to develop an understanding of the behavior of the logarithmic function.</i></p> <p><i>Students will focus on data on the first day, and then on how the function behaves in different situations the next day.</i></p>	<p><i>More applications of Log/Ln functions</i></p> <p><i>Discovering Advanced Algebra (DAA) Chapter 5 Review Questions pg. 296: 10, 11</i></p> <p><i>Progress Check 1 (Animal Populations)</i></p>
<b><i>Lesson Topic: Behavior and Properties of Logarithms</i></b>		
<p><i>What kind of properties do logarithmic functions and expressions have?</i></p> <p><i>How can these properties be used to calculate more efficiently?</i></p>	<p><i>“Properties of Logarithms”</i></p> <p><b><i>Hook:</i></b> <i>Groups will pick random numbers (keeping them small to prevent “overflow” errors) and look at what happens to different combinations of those numbers on the calculator using the logarithmic function. For example, how does <math>\log(a^b)</math> compare to <math>b \cdot \log(a)</math>?</i></p> <p><i>These concepts are critical to success in solving logarithmic equations. Constant</i></p>	<p><i>“Change of Base” worksheet</i></p> <p><i>“Log rules” worksheet</i></p> <p><i>Lesson 5.6: (DAA) Pg. 276 #4, 5</i></p> <p><i>Lesson 5.7: (DAA) Pg. 282 #1-5</i></p> <p><i>Contemporary Mathematics in Context (CMIC) 4A: pg. 169-170 #1-3</i></p> <p><i>Quiz #1: Logarithmic Functions: Properties and definitions</i></p> <p><i>Slide rule Activity pg 280-281</i></p>

<b>Guiding Questions</b>	<b>Instructional Strategies</b>	<b>Checking for Understanding</b>
	<p>reinforcement and practice should be included from this point on in the unit.</p> <p>Reminding students how properties of exponents work (from previous units of study) may help them make connections with how properties of logarithms behave. The “Slide Rule Activity” may be especially beneficial to understanding change of base.</p>	<p>(DAA) as instructional activity OR reinforcement</p>
<b>Lesson Topic: Analyzing Data and Determining Function Type</b>		
<p>How can logarithms be used in analyzing non-linear patterns?</p> <p>How does regression really work on your calculator?</p> <p>How will our knowledge of function transformations help us to write better models for data sets?</p> <p><b>When and why would it be beneficial to represent function families and/or data relationships by re-defining them?</b></p>	<p>“Curve Straightening”</p> <p><b>Hook:</b> Using Fathom software, students will be able to use logs in determining how data is behaving in order to decide whether an exponential, power, or other type of function would best fit (<b>Rehearsal</b>).</p> <p>“Applications of Logarithms” pg. 287-88 Example B or “Cooling Investigation” Transformation of data sets is emphasized, which will be assessed on the quiz.</p>	<p>CMIC 4A pg. 184 #6, On Your Own pg. 185, 189 Pg. 198 #1</p> <p>Lesson 5.8 (DAA) Pg. 290 #6-9</p> <p>Assessment report</p>
<b>Lesson Topic: Inverse Functions</b>		
<p>What is an inverse function?</p> <p>How can you tell if a function has an inverse?</p> <p>How can you tell if an inverse is a function?</p> <p>How will inverse functions help solve problems?</p> <p><b>How is the concept of “undoing” essential to the study of mathematics?</b></p>	<p>“Building Inverses of Functions” Investigation: The inverse</p> <p><b>Hook:</b> Students will use calculators to graph different functions and inverses, then develop equations for the inverse. Emphasis is on how point are related <math>((x,y) \rightarrow (y,x))</math>.</p> <p>“Inverse Symmetry” and “Inverse Composition” will develop other ideas further.</p> <p>Students seem to grasp the concept of an inverse with linear functions rather easily, and the visual representation of an inverse as the reflection</p>	<p>Lesson 5.8 (DAA) #1-10</p> <p>CMIC #4A, Lesson 1 Investigation 1 pg. 142-145 (good graphs, visuals for students who need alternate ways of “seeing” inverses): #1-9 Investigation 2 pg. 147-150 MOREs: pg 150-155 have a variety of questions for all skill levels or topics</p> <p>Inverses worksheets</p> <p>Progress Check: Inverses</p> <p>Quiz #2: Inverse functions and data analysis</p>

<b>Guiding Questions</b>	<b>Instructional Strategies</b>	<b>Checking for Understanding</b>
	over the line $y=x$ . However, it is important to emphasize the composition of functions is the only way to verify an inverse $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$ .	
<b>Lesson Topic: Using Inverse Relations to solve either Logarithmic or Exponential Functions.</b>		
<p>How are exponential and logarithmic functions related?</p> <p>How can exponent/logarithms be used to solve problems?</p> <p><b>How is the concept of “undoing” essential to the study of mathematics?</b></p>	<p><b>(We need 2 activities here- one to emphasize the inverse relationship of exponential and logarithms of matching bases, other to tie into the idea of why log transformations linearize exponential and power relationships.) (Rehearsal)</b></p> <p><i>This lesson’s purpose is to tie together all the ideas from the unit, in addition to practicing the algebraic skills of solving logarithmic and exponential equations.</i></p>	<p>Worksheets: Solving Logarithmic and Exponential equations (multiple versions available for homework, practice and reinforcement purposes)</p> <p>Progress Check 2 (Animal populations)</p> <p>Lesson 5.7: (DAA) Pg. 283 #6-10</p> <p>Lesson 5.8: (DAA) Pg. 289-291 #1-5</p> <p>Exploration (The number e) Pg. 294 Questions 1, 2</p> <p>CMIC 4A: pg. 162 #7-8 Pg. 166-167#5, 7,8</p> <p>CMIC 4B: pg 451-453</p> <p>Unit Review (DAA) Pg. 295-296 # 1-9</p> <p>Unit Test</p>

## **Appendices**

*Text: Discovering Advanced Algebra: An Investigative Approach, Key Curriculum Press*  
(and all additional resources with teacher text)

*Alternate resources: (cited in stage III)*

*Contemporary Mathematics in Context, Book 4A and 4B, Glencoe/McGraw-Hill*

*Miscellaneous Worksheets (attached)*

*Fathom (for either demo or student investigation purposes)*

*Excel (as option for assessment report/data analysis)*

## **Appendix A**

### **Authentic Performance Task Supplied Data**

You have just completed a unit study in your health class learning about the impact HIV virus and the AIDS epidemic in the United States. The devastation caused by HIV/AIDS has stirred up your emotions and you want to do something positive to help people be aware of the problem. After speaking with friends, family and teachers, you decided to raise public awareness in supporting research activities aimed at finding treatment for victims of HIV/AIDS. You strongly feel public funding needs to be increased to support HIV/AIDS research.

You have made it your personal goal to convince public health department in the State of Connecticut to actively support HIV/AIDS reduction research. In order to achieve this goal you have a meeting scheduled with your state representative, who happens to be an influential member of the legislative committee overseeing funding for research activities sponsored by the Department of Health at the Dempsey Hospital. Your representative is passionate about this topic as well and has invited a representative from the Department of Health and your math teacher to attend the meeting. To help you with some of your research your representative has given you data from the *Centers for Disease Control and Prevention* showing the annual number of death resulting from AIDS for the years 1981 through 1994. You recognize the data is not current, but you will use it as a starting point. Your state representative requested that you present a report for each person present and be ready to support its content in a discussion or meeting with the attendees.

To guide along the process, your health teacher (and your math teacher) gave you some suggestions for the report and presentation. One suggestion is to analyze the data several ways.

- Create a scatter plot showing the annual deaths and a scatter plot showing the cumulative total number of deaths.
- Model each of the two sets of data using an appropriate function. Not all models fit all data so be careful.
- For impact, state when the total number of deaths due to AIDS exceed the current population for the State of Connecticut.

Your mentors also remind you, as part of your report and presentation, to

- Describe the behavior of the graph of each model
- Discuss the types of models determined and how they each relate to your goal.
- Discuss how logarithms were used in your analysis/verification
- Discuss how inverse relations are used in your analysis/verification
- Discuss the meanings of each function's variables in relation to what the graph is representing.
- Discuss the mathematical concepts needed to support your conclusions of when the total number of deaths will first exceed the current population for the State of Connecticut.

### **Authentic Performance Task Supplied Data**

<b>Year</b>	<b>Number of Deaths from AIDS</b>
<b>1981</b>	<b>128</b>
<b>1982</b>	<b>460</b>
<b>1983</b>	<b>1,501</b>
<b>1984</b>	<b>3,497</b>
<b>1985</b>	<b>6,961</b>
<b>1986</b>	<b>12,056</b>
<b>1987</b>	<b>16,336</b>
<b>1988</b>	<b>21,040</b>
<b>1989</b>	<b>27,691</b>
<b>1990</b>	<b>31,402</b>
<b>1991</b>	<b>36,307</b>
<b>1992</b>	<b>40,516</b>
<b>1993</b>	<b>42,992</b>
<b>1994</b>	<b>46,050</b>

United States Centers for Disease Control and Prevention, *HIV/AIDS Surveillance Report*, Year-End Edition 1995

## Appendix B

Precalculus  
Unit 3: Logarithmic & Inverse Functions  
Exam

Name \_\_\_\_\_

Period \_\_\_\_\_

- Which of the following is NOT ALWAYS true about a linear function and its inverse?
  - If  $g(a)=b$ , then  $g^{-1}(b)=a$ .
  - The lines are perpendicular to each other.
  - They reflect across the line  $y=x$ .
  - $(g \circ g^{-1})(x) = x$
  - They “undo” each other.
- Which of the following would be the next step in solving the logarithmic equation:  
 $\log_2(x-1) = -3$ 
  - $\log_2(x) - \log_2(1) = -3$
  - $x-1 = \frac{-3}{\log_2}$
  - $2^{x-1} = -3$
  - $2^{\log_2(x-1)} = 2^{-3}$
  - $2^x - 2^1 = -3$
- Solve the equation for x:  $6 = 3e^{2x}$ 
  - $e \ln(2)$
  - $\frac{1}{2} \ln(2)$
  - $\frac{1}{2} \ln(-2)$
  - $2 \ln\left(\frac{1}{2}\right)$
  - $\frac{\sqrt{2}}{e}$
- Solve the equation for x:  $\log_2(x+3) - \log_2(8x) = 2$ 
  - 0
  - .097
  - 4.13
  - .256
  - none of the above
- In which of the following equations is x **not** equal to 1?
  - $x = \log 10$
  - $x = \ln e$
  - $\log_2 x = 0$
  - $\log_x 4 = 2$
  - none of the above

6.  $\log_b(mn)^x$  could be rewritten as

- a.  $x(\log_b m + \log_b n)$
- b.  $m\log_x b + n\log_x b$
- c.  $x \frac{\log_b m}{\log_b n}$
- d.  $\log(mn) + \log(nx)$
- e.  $b\log_x n - b\log_x m$

7. You have just worked through a logarithm problem and obtained the following result for  $n$ :

$$n = \log_{12}(3.5)$$

If you want to use your calculator to obtain a decimal approximation, which of the following expressions could be used?

- a.  $\frac{\log_{10}(3.5)}{\log_{10}(12)}$
- b.  $\frac{\log_{10}(3.5)}{\log_{10}(6) + \log_{10}(2)}$
- c.  $\frac{\ln(3.5)}{\ln(12)}$
- d. all of the above
- f. none of the above

8. A fruit fly colony grows continuously at the rate of 4.5 % per day. If the population begins at 300, what equation gives the answer to the question “how long will it take the fruit fly population to reach 850?”?

- a.  $300 = 850e^{.045t}$
- b.  $t = \frac{\ln 850}{\ln 300}$
- c.  $.045(\ln\left(\frac{850}{300}\right)) = t$
- d.  $850 = 300\left(1 + \frac{.045}{365}\right)^{365t}$
- e.  $\left(\ln\left(\frac{850}{300}\right)\right)\left(\frac{1}{.045}\right) = t$

9. Simplify the following exponential expression:

$$\frac{4w^{-2}x^3y^{-5}z^{\frac{0}{-2}}}{12w^2x^{\frac{1}{2}}y^{-1}z^{-3\frac{1}{0}}}$$

- a.  $\frac{9x^3y^{12}z^4}{w^4y^4}$
- b.  $9w^{-4}x^{\frac{5}{2}}y^{-4}z^4$
- c.  $\frac{9w^8y^8}{x^5z^8}$
- d.  $\frac{9\sqrt{x^5}}{w^4y^4z^4}$
- e. none of the above
10. Given:  $\log_a(4) = -2.0588$ ,  $\log_a(3) = -1.6316$ , which of the following is false?
- a.  $\log_a(16) = -4.1176$
- b.  $\log_a(12) = -3.6904$
- c.  $\log_a(27) = -4.8948$
- d.  $\log_a\left(\frac{1}{3}\right) = 1.6316$
- e.  $\log_a(\sqrt{4}) = 1.5638$
11. The amount of energy produced by an earthquake (E, in joules) is used to determine the magnitude reading on the Richter scale using the function:
- $$M(E) = \frac{2}{3} \log\left(\frac{E}{10^{4.4}}\right)$$
- How many times greater is the energy released in a magnitude 7 earthquake than a magnitude 4 earthquake?
- a. 10 times
- b. 123 times
- c. 305 times
- d. 5561 times
- e. 31,623 times

12. According to a recent Hartford Courant article, an emerging industry in Connecticut is wine-making. Find the equation that best fits the data below.

Let the original year be 1993. Let  $n$  = the number of years after 1993. Let  $P(n)$  = the annual wine production (in cases).

Year	# of Cases Produced
1993	17,000
1994	19,600
1995	22,600
1996	26,100
1997	30,000

- a.  $P(n) = 17000(1.15)^n$
- b.  $P(n) = 30000(1.15)^n$
- c.  $P(n) = 30000 + 3000n$
- d.  $P(n) = 30000(1.2)^n$
- e.  $P(n) = 17000 + 5000n$

13. The perceived magnitude (brightness) of two stars can be modeled by the function  $m_1 - m_2 = 2.5 \log\left(\frac{I_2}{I_1}\right)$ , where  $m_1$  &  $m_2$  are the magnitudes and  $I_1$  &  $I_2$  are the intensities of the two stars. If one star has a light magnitude of zero with an intensity of  $2.48 \times 10^{-8}$  watts per square meter, how intense is the other star with a magnitude of 3?

- a.  $2.48 \times 10^{-13}$     b.  $1.57 \times 10^{-9}$     c.  $3.93 \times 10^{-7}$     d. .00248    e. no solution

14. What is the inverse of the function:  $g(x) = \frac{2(x-3)}{5}$  ?

- a.  $g^{-1}(x) = 5(x+6)$
- b.  $g^{-1}(x) = \frac{5(x+6)}{2}$
- c.  $g^{-1}(x) = \frac{2(x-3)}{5}$
- d.  $g^{-1}(x) = \frac{5}{2}x + 3$
- e.  $g^{-1}(x) = 2.5x - 3$

15. Which of the following has an inverse that is also a function?

a.

x	f(x)
-3	5
-1	-1
0	-1
1	1
3	4

b.

x	f(x)
10	-10
15	-8
6	-3
42	0
4	4

c.

x	f(x)
5	5
27	-18
8	-8
15	-15
0	8

d. a and b only

e. a and c only

16. The graph of the function  $g(x) = \log_3(x + 4)$  has a domain and range of

a. domain  $(-3, \infty)$  and range  $(-\infty, \infty)$

b. domain  $(-4, \infty)$  and range  $(-\infty, \infty)$

c. domain  $(3, \infty)$  and range  $(-\infty, \infty)$

d. domain  $(4, \infty)$  and range  $(-3, 4)$

e. None of the above

## Appendix C

Precalculus

Name \_\_\_\_\_

Unit 3: Logarithmic and Inverse Functions

Date \_\_\_\_\_

Quiz #1

1. You take out a \$60,000 school loan. The time  $t$  in months that it takes to pay off the loan at 9% annual interest with monthly payments of  $x$  dollars is given by

$$t = 133.83 \ln\left(\frac{x}{x - 450}\right)$$

You start paying back this loan after you graduate college.

- a) Estimate the length (term) of the \$60,000 loan you will have in order to pay off the loan if your monthly payments are \$700.
- b) What would the monthly payment be you want to pay off the \$60,000 loan in five years.
- c) You want to draw a graph of this relation to determine if there are any other payback options. In order to do this, you need to determine the domain and range for the relation. Determine the domain and range for this relation and sketch a well-dressed graph showing where the solutions to parts a) and b) are on the graph.
2. As a pharmacologist, you are aware of the absorption rates of drugs administered intravenously for pain control. You know that for one such medication the function  $f(t) = 80 - 23 \ln(1 + t)$  for the time interval  $0 \leq t \leq 24$ , models the amount of drug in the body after  $t$  hours.
- a) What is the initial dosage of the medication administered?
- b) How much of the medication is present two hours after initial administration?
- c) Draw a graph of this function stating its domain and range and explaining what they represent.

Use the rules of the logarithms and the following for problems 3 -7. Show your reasoning.

$$\log_a 2 = .553$$

$$\log_a 3 = .877$$

$$\log_a 5 = 1.285$$

3.  $\log_a 10 =$

4.  $\log_a \frac{3}{5} =$

5.  $\log_a 9 =$

6.  $\log_a \frac{1}{2} =$

7.  $\log_a \sqrt{3} =$

8. Using properties of logarithms, explain how to find the solution to the following problem:  
 $\log_4 15 = x$

9. What are the similarities and differences between  $f(x) = \log x$  and  $g(x) = \ln x$ ? Be sure to include information on domain/range, function behavior, and properties.

## Appendix D

Precalculus

Name \_\_\_\_\_

Unit 3: Logarithmic and Inverse Functions

Date \_\_\_\_\_

Quiz #2

1. In an experiment, water was heated and put in a cool location. Then, the temperature was measured at periodic intervals for the rest of the evening. A table of the results is below.

Time (minutes)	Temperature (°F)	
0	124	
5	118	
16	114	
20	109	
35	106	
50	97	
65	89	
85	82	
128	74	
144	64	
178	62	
208	59	
244	55	
299	50	
331	49	
391	47	

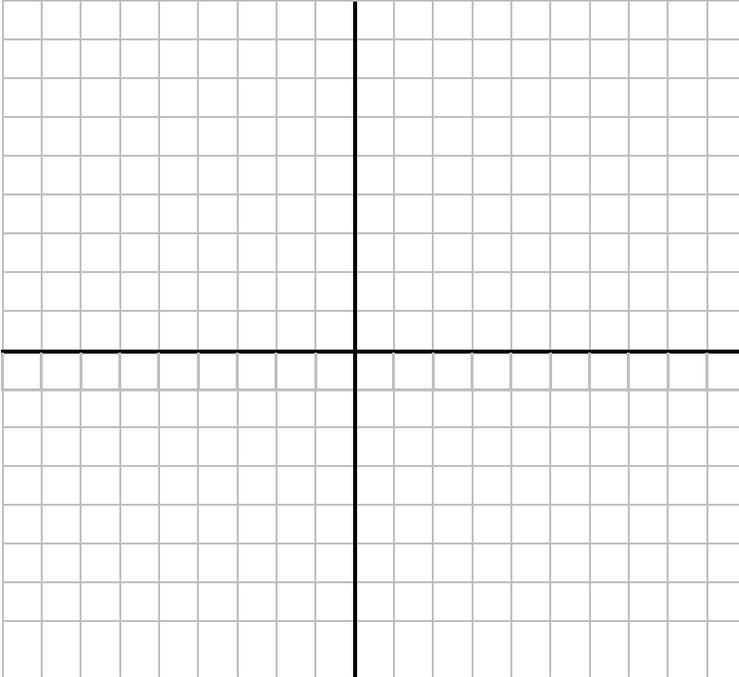
- a. What type of function appears good model for this data? Give reasons to support your answer, explain any transformation that you did to the data to confirm this.
- b. Create a scatterplot of (time, temperature) data and determine the best fit equation. Explain how you know the equation you graphed is the best fit.
- c. Using your regression equation, determine when the water temperature would have been room temperature (about 72°F). Are you confident in this answer? Explain.
- d. Determine the temperature of the water after 24 hours. How confident are you in this answer? Explain.

2. These functions are inverses of each other:

$$f(x) = \frac{1}{2}x - 2$$

$$f^{-1}(x) = 2x + 4$$

- a. Graph both functions and explain how the graph shows that the functions are inverses.



3. Determine the inverse of each of the following functions. Show or explain your reasoning.

Explanation:

- b. Show that the functions are inverses **algebraically**.  
a.  $f(x) = \frac{3}{4}x - 5$

b.  $g(x) = (x - 3)^2$

c.  $(-2, 5), (0, 1), (1.5, -3), (3, -3), (5, -5)$

## Appendix E

Precalculus  
Unit 3: Logarithmic & Inverse Functions  
Progress Check 1

Name \_\_\_\_\_  
Date \_\_\_\_\_

The population of an animal species introduced into an area sometimes increases rapidly at first and then more slowly over times. A logarithmic function models this kind of growth. Suppose that a population of  $N$  deer in an area  $t$  months after the deer are introduced is given by the equation  $N=325 \log (4t+2)$ .

- a. Use this model to predict the deer population after
  - 3 months
  - 6 months
  - 12 months
  - 18 months
- b. According to the model, how long will it take for the population to reach 800?
- c. Describe what this model tells us about the long-term population of the deer.

Precalculus  
Unit 3: Logarithmic & Inverse Functions  
Progress Check 2

Name \_\_\_\_\_  
Date \_\_\_\_\_

The population of an animal species introduced into an area sometimes increases rapidly at first and then more slowly over times. A logarithmic function models this kind of growth. Suppose that a population of  $N$  deer in an area  $t$  months after the deer are introduced is given by the equation  $N=325 \log (4t+2)$ .

- a. According to the model, how long will it take for the population to reach 800?  
Show/explain how you can find the solution algebraically.
- b. Are there any limitations on the model? Explain.
- c. Describe a reasonable practical domain and range that take into account your limitations.

## Appendix F

Precalculus  
Unit 3, Logarithmic & Inverse Functions  
Progress Check

Name \_\_\_\_\_

Date \_\_\_\_\_

$$f(x) = \frac{1}{2}x + 3$$

1. Find the inverse of the following function:

$$f^{-1}(x) =$$

2. How can you show that the functions are inverses graphically?
3. How can you tell from a table of values that the functions are inverses?
4. How can you use composition to verify that the functions are inverses?
5. What are inverses good for? Why should we study them?

## Appendix G

Precalculus  
Unit 3, Logarithmic & Inverse Functions  
Homework

Name \_\_\_\_\_  
Thursday, December 15, 2005

I. Solve the following equations algebraically.

1. $4 \cdot 2^x = 10$	2. $\frac{e^{2x-1} + 2}{3} = 1$
3. $\frac{\log_5(3x-4)}{2} = 3$	4. $\log_3 \frac{x}{3} = 7$
5. $5e^{x-1} = 17$	6. $\sqrt{\log_4(x)} - .25 = 0$
7. $e^{\frac{3x+1}{2}} = 6$	8. $4 \ln(1-2x) = .75$
9. $5^{\frac{2x+7}{3}} = 2$	10. $\frac{\log_{0.5}(3)}{\log_{0.5}(4x)} = \frac{2}{3}$

II. Solve the following algebraically.

1. Suppose a certain radioactive material decays at a rate of 3 % a day. You begin with 10 grams of the material. Write the function describing the amount of material left in grams ( $A(n)$ ) remaining after  $n$  days.

- At how many days is half the amount of material remaining?
- What equation did you actually solve to answer that question?

## Appendix H

Precalculus  
Unit 3: Logarithmic & Inverse Functions

Name \_\_\_\_\_  
Date \_\_\_\_\_

### Magnitude of Disaster: The Richter Scale

The Richter scale was developed in 1935 in order to study the large number of earthquakes that occurred in California. Charles Richter and Beno Gutenberg of the California Institute of Technology wanted to be able to distinguish the intensity of smaller earthquakes, since they occurred more frequently than large earthquakes.

The Richter scale has no upper or lower limit, and thanks to advances in technology, seismographs can record earthquakes with negative magnitudes. Micro earthquakes (ones with Richter readings less than 2.0) occur about 8,000 times per day!

Below is a table of 5 “micro” earthquakes at the Kilauea Caldera (Hawaii) in 1959 (from the National Geophysical Data Center, [www.ngdc.noaa.gov](http://www.ngdc.noaa.gov))

Energy (in joules)	Magnitude
118748	.5
469507	.9
662057	1.0
933575	1.1
1316445	1.2

1. Make a scatterplot of the (Energy, Magnitude) data. Describe any patterns that you see. How is this data different from others we studied?
2. What parent function do you think might be a good model for this data? Give reasons to support your answer.
3. In order to help you determine the function, add this data to your scatterplot.

Energy (in joules)	Magnitude
1000	-.89
10000	-.22
22000	.009
52000	.26
82000	.39

What does this additional data tell you about the parent function? Describe what you see.

4. What transformations on the parent function will fit this data? Write a function that fits this data.

$M(j) =$  \_\_\_\_\_

5. What made this function hard to fit? How does this relate to our earlier study of parent functions?

6. What is the domain and range of your function? Give reasons to justify your answer.

Domain:

Range:

7. Is this function discrete or continuous? Justify your response.

Earlier, we developed a Richter scale function ( $M(j)=-2.9+.29\ln(j)$ ). Another commonly accepted function to determine the magnitude of an earthquake is  $M(j)=-2.9+.67\log(j)$ .

1. Are these two functions equivalent? Justify your response. Why might they both be true?

*Use one of the functions to determine the answers to the following.*

2. If an earthquake releases  $10^{13}$  joules of energy, what is its magnitude on the Richter scale?
3. A 2 on the Richter scale is hardly noticeable, while a 5 is capable of shattering windows and is considered "minor". How many joules do each of these produce? What does this tell you about the intensity of an earthquake?
4. The San Fransisco earthquake of 1906 registered an 8.3 on the Richter scale. The San Fransisco earthquake of 1989 registered 7.1. How do these two earthquakes compare in intensity?
5. If an earthquake is twice as intense as the 1989 earthquake, what is its magnitude? Ten times? A hundred times?

*Sound Intensity*

6. In order to calculate loudness level in decibels, physicists measure the intensity of a sound compared to the threshold of human hearing ( $10^{-12}$  watts/square meters) using the following equation:

$$L = 10 \log\left(\frac{I}{10^{-12}}\right)$$

An ordinary conversation  $10^{-6}$  W/m<sup>2</sup>. How many decibels is this?



## Appendix I

Precalculus

name:

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### Unit 3 – Inverses and Logarithmic Functions

If  $\log_a 2 = .356$  and  $\log_a 3 = .565$  and  $\log_a 5 = .827$ , use the rules of logs to determine:  
(show your calculations)

$$\log_a 6 =$$

$$\log_a 9 =$$

$$\log_a 10 =$$

$$\log_a 30 =$$

$$\log_a \frac{2}{5} =$$

$$\log_a 1.5 =$$

$$\log_a 30 =$$

$$\log_a 25 =$$

$$\log_a \frac{2}{3} =$$

$$\log_a 3^4 =$$

$$\log_a \sqrt{3} =$$

$$\log_a 2\sqrt{5} =$$

If  $\log_a 2 = .356$  and  $\log_a 3 = .565$  and  $\log_a 5 = .827$ , use the rules of logs to determine:

$.356 + .567$  written as a log is:

$.827 - .356$  written as a log is:

$3 * .356$  written as a log is:

$\frac{1}{2} * .565$  written as a log is:

$.565 + 2 * .827$  written as a log is:

$.356 - .356$  written as a log is:

$.565 - .565$  written as a log is:

$.356 + .565 + .827$  written as a log is:

$.827 + .827$  written as a log is:

$2 * .827$  written as a log is:

Using your calculator, determine (and show how you did that):

$$\log_4 7$$

$$\log_7 57$$

$$\log_6 12$$

$$\log_8 64$$

## Appendix J

Precalculus  
Unit 3, Logarithms & Inverse Functions

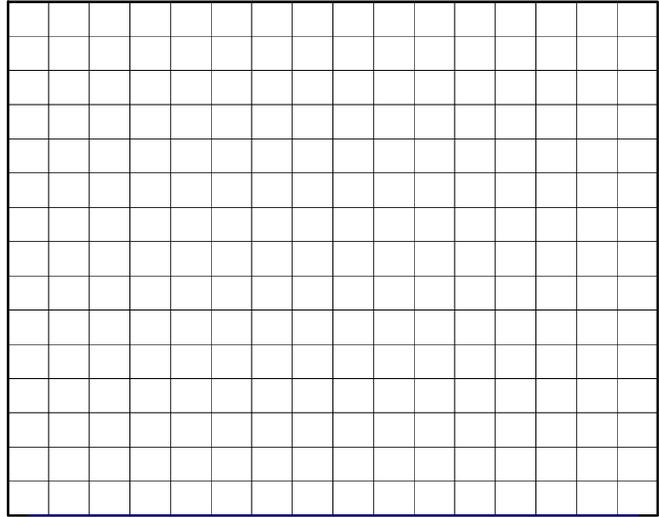
name \_\_\_\_\_  
December 1, 2005

1. Graph and label the following ordered pairs ( x, y):  
 A ( -5, -2)  
 B ( -3, 0)  
 C ( -1, 1)  
 D ( 0, 2)  
 E ( 4, 3)

Is it a function?

Call it f(x).

2. Swap the x and y and graph those pairs ( y, x) and label them as A', B', C', etc.



Is it a function?

Call it g(x).

3. What is  $(f \circ g)(-2)$  ?

What is  $(f \circ g)(3)$  ?

What is  $(g \circ f)(-3)$  ?

What is  $(g \circ f)(0)$  ?

What is  $(f \circ g)(0)$  ?

What is  $(g \circ f)(4)$  ?

4. In every case you should have found that  $(f \circ g)(a) = a$  and  $(g \circ f)(a) = a$ .

Therefore we can state that g(x) and f(x) are \_\_\_\_\_ of each other.

In List 1 in your calculator, place the integers -5 to 5.

In List 2, place the equation  $2L1-1$ . Use Stat Plot to plot your points.

What type of function is it? What is the regression equation of it? Graph it.

Place List 1 in L4 and List 2 in L3. Use Stat Plot 2 to graph the points ( L3, L4).

5. Give two different reasons why we can call those two equations inverses of each other.

5. Repeat the process you performed in # 4 and this time make the function in  $L2 = -.5L1 + 2$  (which is really  $y = -.5x + 2$ ). Use the same process to find the equation of the inverse function.

Call the first function  $h(x) = -.5x + 2$

Call the second function  $j(x)$

Perform these compositions:

$$(h \circ g)(7) =$$

$$(h \circ g)(4) =$$

$$(g \circ h)(-2) =$$

$$(h \circ g)(20.8) =$$

$$(g \circ h)(16) =$$

$$(h \circ g)(\pi) =$$

$$(h \circ g)\left(\frac{1}{5}\right) =$$

$$(g \circ h)(x) =$$

6. Find the inverses of these functions:

$$k(x) = 3x + 12$$

$$m(x) = \frac{2}{5}x - 5$$

$$r(x) = -4x + 3$$

## Appendix K

	4 EXCEEDS STANDARD	3.2 MEETS STANDARD	2.5 NEAR STANDARD	1 BELOW STANDARD
<b>1. Completeness</b>	<ul style="list-style-type: none"> <li>Includes four trig functions, including Boolean algebra</li> <li>complete algebraic solution to Part 2, including consideration of context</li> </ul>	<ul style="list-style-type: none"> <li>Includes four trig functions</li> <li>complete algebraic solution to Part 2</li> </ul>	<ul style="list-style-type: none"> <li>Includes at least two trig functions</li> <li>Some evidence of algebraic reasoning in Part 2</li> </ul>	Major components inadequate or missing
<b>2. Correct Math</b>	Solutions are correct conceptually and contain NO arithmetic errors.	Solutions are correct conceptually but may contain SOME arithmetic errors.	Some evidence of conceptual understanding and contains some arithmetic errors.	Major conceptual misunderstanding and/or arithmetic errors.
<b>3. Supporting Evidence</b>	<ul style="list-style-type: none"> <li>Graphs presented with complete labeling</li> <li>Graphs imbedded in text, rather than at the end of paper</li> <li>Diagram illustrating relevant information</li> </ul>	<ul style="list-style-type: none"> <li>Graphs presented with mostly complete labeling</li> <li>Diagram illustrating relevant information</li> </ul>	Graphs and diagrams presented but may be inaccurate or incomplete	Graphs and/or diagrams missing
<b>4. Communication</b>	Exemplary organization. Paper could be used as a teaching tool.	Logical and well-written. Procedures clearly explained	Logical structure but key details are missing	Little organization. Difficulty with logical flow.
<b>5. Function Descriptors</b>	<ul style="list-style-type: none"> <li>Variables defined and function notation used</li> <li>Continuity addressed</li> <li>Domain and range values are correct and notation is correct.</li> </ul>	<ul style="list-style-type: none"> <li>Variables defined and function notation used</li> <li>Continuity addressed</li> <li>Domain and range may contain minor error in notation and/or endpoints</li> </ul>	<ul style="list-style-type: none"> <li>Concepts inadequately addressed</li> <li>Contains conceptual errors</li> </ul>	Concepts missing
<b>6. Introduction</b>	The purpose of the paper is clearly explained with all key elements. Attention paid to restating the problem in an interesting and engaging way	The purpose of the paper is clearly explained with all key elements.	Introduction included but fails to clearly explain key elements in the paper.	Introduction missing.
<b>7. Presentation</b>	<ul style="list-style-type: none"> <li>Attention paid to visual impact of paper</li> <li>Effective use of Word, Mathtype, Excell, TI-84, or other software</li> <li>Grammar, syntax, and spelling are correct</li> </ul>	<ul style="list-style-type: none"> <li>Attention paid to visual impact of paper</li> <li>Mostly done electronically, but formulas may be handwritten</li> <li>Grammar, syntax, and spelling are mostly correct</li> </ul>	<ul style="list-style-type: none"> <li>Some attention paid to visual impact of paper</li> <li>Major portions of paper handwritten</li> <li>Errors in grammar, syntax, and/or spelling</li> </ul>	Paper is disorganized, handwritten, and may contain errors in grammar, syntax, and/or spelling
I. EXTRAS	<ul style="list-style-type: none"> <li>Use of technology +5 max</li> </ul>			<ul style="list-style-type: none"> <li>Turned in late -10 * days</li> </ul>

1. Completeness 2. Correct Math 3. Supporting Evidence 4. Communication 5. Function Descriptors 6. Introduction 7. Presentation	_____ x 3 _____ x 5 _____ x 3 _____ x 4 _____ x 2 _____ x 1 _____ x 1	Raw Score  _____ 76	% Score _____  Extras _____  Final Grade _____
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## **Appendix L**

## **Appendix M**